Software implementation and testing of GARCH models

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Abstract: This paper describes the software implementation of GARCH routines in Fortran 77. The routines considered here cover both symmetric and asymmetric GARCH and also shocks having Gaussian and non-Gaussian distributions. Extensive examples of using the software are provided and the test results from Monte Carlo simulations are presented.

Keywords: GARCH, ARCH, maximum likelihood estimation, volatility, generalised autoregressive heteroskedasticity, asymmetry, Fortran 77

1 Introduction

The modelling of sequences with time-dependent variance is crucial to many areas of mathematical finance. This technical report describes the software implementation and testing of a set of univariate generalised autoregressive conditional heteroskedastic (GARCH) routines that have been developed at NAG Ltd for its numerical libraries. Although we are primarily concerned with GARCH software that has been developed for the next release of the NAG Fortran 77 Library [1], some GARCH software is already contained in the current version of the NAG C Library [2]. The Fortran 77 software described here can be used for GARCH(*p,q*) models with arbitrary values of *p* and *q*. There are routines for the generation of GARCH sequences, model estimation and volatility forecasting. The estimation routines return not only the parameter estimates but also other important statistics such as: the standard errors, the scores and the value of the log-likelihood function for the calculated model parameters.

Other capabilities of the software include the following:

- regression-GARCH(*p,q*) models
- symmetric models
- asymmetric models
- shocks with Gaussian and non-Gaussian distributions

Since both the NAG Fortran 77 and C libraries are implemented as Dynamic Link Libraries (DLLs) the GARCH software can be easily used from the Microsoft Windows environment. This means that the GARCH routines can be readily incorporated into Microsoft software such as Excel, Visual Basic, etc.

2 The GARCH models

The standard (symmetric) regression-GARCH(*p,q*) model [3] [4] [5], with Gaussian shocks e_t, takes the following form:

$$y_t = b_0 + x_t^T b + e_t, \quad e_t \mid \psi_{t-1} = N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{t-1}^2 + \sum_{j=1}^{p} \beta_i h_{t-j}$$

This process is described by $q+1$ coefficients $\alpha_i, i = 0,\dots,q,$ p coefficients β_i $, i = 1,\dots,p,$ mean b_0, k linear regression coefficients $b_i, i = 1,\dots,k,$ endogenous/exogenous variables y_t and x_t respectively, shocks e_t, conditional variance h_t, and the set of all information up to time *t*, ψ_t.

It should be noted that for p = 0 a GARCH(p,q) model is also called an ARCH(q) model.

Empirical studies on financial time series have shown that they are characterised by increased conditional variance h_t following negative shocks (bad news). The distribution of the shocks have been also found to exhibit considerable leptokurtosis. Since the standard Gaussian GARCH model cannot capture these effects various GARCH model extensions have been developed [6].

The asymmetric GARCH models considered here are:

AGARCH(p,q)-type1

$$h_t = a_0 + \sum_{i=1}^{q} a_i (e_{t-i} + g)^2 + \sum_{j=1}^{p} b_i h_{t-j}$$

AGARCH(p,q)-type2

$$h_t = a_0 + \sum_{i=1}^{q} a_i (|e_{t-i}| + g e_{t-i})^2 + \sum_{j=1}^{p} b_i h_{t-j}$$

GJR-GARCH(p,q), or Glosten, Jagannathan and Runkle GARCH [7]

$$h_t = a_0 + \sum_{i=1}^{q} (a_i + g S_{t-1}) e_{t-i}^2 + \sum_{j=1}^{p} b_i h_{t-j}$$

where $S_t = 1$ if $e_t < 0$ and $S_t = 0$ if $e_t \geq 0$

EGARCH(p,q), or exponential GARCH

$$\ln(h_t) = a_0 + \sum_{i=1}^{q} a_i z_{t-i} + \sum_{i=1}^{q} f_i \left(|z_{t-i}| - \mathrm{E}\left[|z_{t-i}| \right] \right) + \sum_{j=1}^{p} b_i \ln(h_{t-j})$$

where $z_t = \dfrac{e_t}{\sqrt{h_t}}$ and $\mathrm{E}[|z_{t-i}|]$ denotes the expected value of $|z_{t-i}|$

In all these models the shocks, e_t, can either have a Gaussian distribution or Student's *t*-distribution with a specified number of degrees of freedom.

In AGARCH-type1 the asymmetric effects are modelled via the extra parameter g . For example, in the standard GARCH(1,1) model when h_{t-1} is fixed $h_t = h(e_{t-1})$ is a parabola with a minimum at $e_{t-1} = 0.$ The introduction of the additional parameter g shifts the parabola horizontally so that the minimum occurs at $e_{t-1} = -g$. The conditional variance following negative shocks can therefore be enhanced by choosing $g < 0,$ so that $h(-e_{t-1}) > h(e_{t-1})$ for $e_{t-1} > 0.$

In an AGARCH-type2 model the inclusion of g can also result in an enhancement of h_t following a negative shock e_{t-1}. For a GARCH(1,1) model $h(-e_{t-1}) > h(e_{t-1})$ for $e_{t-1} > 0$ and $g < 0.$

Similarly in the GJR-GARCH(1,1) model the value of h_t is increased above the symmetric case when $e_{t-1} < 0$ and $g > 0.$

For EGARCH, asymmetric response arises from the terms $\sum_{i=1}^{q} a_i z_{t-i}$. In an EGARCH(1,1), if $a_1 < 0$ then a negative shock $e_{t-1} < 0$ increases the value of h_t, that is $\ln\{h(-z_{t-1})\} > \ln\{h(z_{t-1})\}$.

All the GARCH processes above are uniquely described by the parameter vector q, where $q = (b_0, b^T, w^T)$, $w^T = (a_0, a_1, a_2, \ldots, a_q, b_1, b_2, \ldots, b_p, g)$ and $b^T = (b_1, \ldots, b_k)$.

The GARCH model implementations here all rely on finding the value of q which maximises the conditional log-likelihood (objective) function

$$lf = \frac{1}{2} \sum_{i=1}^{T} \left(\log(h_i) + \frac{e_i^2}{h_i} \right)$$

where T is the number of terms in the sequence.

This is achieved by starting with an initial approximation for q and then using numerical optimisation to iterate to an acceptable solution. In all the GARCH estimation routines, apart from EGARCH, an analytic expression for the Jacobian of the objective function was used for the optimisation stage.

The standard errors for the parameter estimates can be computed by using the well known result [8] that the maximum likelihood estimate for q is asymptotically normal with mean q and covariance matrix $\Im^{-1}$ where $\Im$ (The Fisher Information Matrix) is given by:

$$\Im = \mathrm{E}\left[\sum_{i=1}^{T} \frac{\partial^2 lf}{\partial q \, \partial q^T} \right]$$

3 The GARCH software

Here we list the available software for sequence generation, model parameter estimation, and forecasting of regression-GARCH(*p,q*) sequences.

3.1 Generation

The following routines generate a given number of terms from various symmetric and asymmetric GARCH(*p,q*) sequences.

Specification

AGARCH-type1

```
SUBROUTINE G05HKF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, YT, FCALL, RVEC, IFLAG)
```

AGARCH-type2

```
SUBROUTINE G05HLF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, YT, FCALL, RVEC, IFLAG)
```

GJR-GARCH

```
SUBROUTINE G05HMF (DIST, NUM, IP, IQ, THETA, GAMMA, DF, HT, YT, FCALL, RVEC,IFLAG)
```

EGARCH

```
SUBROUTINE G05HNF (DIST, NUM, IP, IQ, THETA, DF, HT, YT, FCALL, RVEC, IFLAG)
```

The routine parameters have the following meanings:

Parameters

DIST - CHARACTER*1.

On entry, the type of distribution to use for e_i
if *DIST* = 'N' then a Normal distribution is used.
if *DIST* = 'T' then a Student's *t*-distribution is used.

NUM - INTEGER
On entry, the number of terms in the sequence, *T*

IP - INTEGER.
On entry, the number of moving average coefficients, *p*

IQ - INTEGER.
On entry, the number of auto-regressive coefficients, *q*

THETA- DOUBLE PRECISION array.
On entry, the parameters of the GARCH model

GAMMA - DOUBLE PRECISION.
On entry, the asymmetry parameter g for the GARCH sequence.

DF - DOUBLE PRECISION
On entry, the number of degrees of freedom for the Student's *t*-distribution.
It is not referenced if *DIST* = 'N'

HT - DOUBLE PRECISION array.

On exit, the conditional variances for the GARCH sequence, $h_i, i = 1, \ldots, T$

YT - DOUBLE PRECISION array.

On exit, the observations for the GARCH sequence, $e_i, i = 1, \ldots, T$.

FCALL - LOGICAL.
On entry, if *FCALL = .TRUE.* then a new sequence is to be generated,
else a given sequence is to be continued using the information in RVEC.

RVEC - DOUBLE PRECISION array.
On entry, the array contains information required to continue a sequence if *FCALL = .FALSE*.
On exit, contains information that can be used in a subsequent call, with *FCALL = .FALSE*.

IFLAG - INTEGER.
The error indicator

3.2 Estimation

The following routines estimate the model parameters of various symmetric and asymmetric regression-GARCH(*p,q*) sequences.

Specification

AGARCH-type1

```
SUBROUTINE G13FAF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, ISYM, THETA, SE, SC, COVAR, LDC, HP,
                   ETM, HTM, LGF, COPTS, MAXIT, TOL, WORK, LWORK, IFLAG)
```

AGARCH-type2

```
SUBROUTINE G13FCF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM,
                   LGF, COPTS, MAXIT, TOL, WORK, LWORK, IFLAG)
```

GJR-GARCH

```
SUBROUTINE G13FEF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM,
                   LGF, COPTS, MAXIT, TOL, WORK, LWORK, IFLAG)
```

EGARCH

```
SUBROUTINE G13FGF (DIST, YT, X, LDX, NUM, IP, IQ, NREG, MN, THETA, SE, SC, COVAR, LDC, HP, ETM, HTM,
                   LGF, COPT, MAXIT, TOL, WORK, LWORK, IFLAG)
```

The routine parameters have the following meanings:

Parameters

DIST - CHARACTER*1

On entry, the type of distribution to use for e_i. If *DIST* = 'N' then a Normal distribution is used, if *DIST* = 'T' then a Student's t- distribution is used.

YT - DOUBLE PRECISION array.

On entry, the sequence of observations, $e_i, i = 1, \ldots, T$.

X - DOUBLE PRECISION array.

On entry, the ith row of *X* contains the time dependent exogenous vector, $x_i, i = 1, \ldots, T$.

LDX - INTEGER.
On entry, the first dimension of the array *X*

NUM - INTEGER.
On entry, the number of terms in the sequence, T

IP - INTEGER.
On entry, the number of moving average coefficients, *p*

IQ - INTEGER.
On entry, the number of auto-regressive coefficients, *q*

NREG - INTEGER.
On entry, the number of regression coefficients, *k*

MN - INTEGER.

On entry, if *MN* = 1 then the mean term b_0 will be included in the model

ISYM - INTEGER.
On entry, *if ISYM* = 1 then the asymmetry term g will be included in the model (this only applies to G13FAF)

THETA - DOUBLE PRECISION array.

On entry, the initial parameter estimates for the vector q

On exit, the estimated values $\hat{q}$ for the vector q

SE - DOUBLE PRECISION array.

On exit, the standard errors for $\hat{q}$

SC - DOUBLE PRECISION array.

On exit, the scores for $\hat{q}$

COVAR - DOUBLE PRECISION array.

On exit, the covariance matrix of the parameter estimates $\hat{q}$, that is the inverse of the Fisher Information Matrix.

LDC - INTEGER.
On entry, the first dimension of the array *COVAR*

HP - DOUBLE PRECISION
On entry, if *COPTS(2) = .FALSE.* then *HP* is the value to be used for the pre-observed of the conditional variance.
If *COPTS(1) = .TRUE.* then *HP* is not referenced.

On exit, if *COPTS(2) = .TRUE.* then *HP* is the estimated value of the pre-observed of the conditional variance.

ET - DOUBLE PRECISION array.

On exit, the estimated residuals, $e_i, i = 1, \ldots, T$.

HT - DOUBLE PRECISION array.

On exit, the estimated conditional variances, $h_i, i = 1, \ldots, T$

LGF - DOUBLE PRECISION.

On exit, the value of the likelihood function at $\hat{q}$

COPTS - LOGICAL array
If *COPTS(1) = .TRUE.* then stationary conditions are enforced, otherwise they are not.
If *COPTS(2) = .TRUE.* then the routine provides initial parameter estimates of the regression terms, otherwise these are provided by the user.

MAXIT - INTEGER.
On entry, the maximum number of iterations to be used by the optimisation routine when estimating the GARCH parameters.

TOL - DOUBLE PRECISION
On entry, the tolerance to be used by the optimisation routine when estimating the GARCH parameters.

WORK - DOUBLE PRECISION array, workspace.

LWORK - INTEGER.
On entry, the size of the work array WORK.

IFLAG - INTEGER.
The error indicator.

3.3 Forecasting

The following routines compute the volatility forecast for various symmetric and asymmetric GARCH(*p,q*) sequences.

Specification

AGARCH-type1
SUBROUTINE G13FBF (NUM, NT, IP, IQ, THETA, GAMMA, CVAR, HT, ET, IFLAG)

AGARCH-type2
SUBROUTINE G13FDF (NUM, NT, IP, IQ, THETA, GAMMA, CVAR, HT, ET, IFLAG)

GJR-GARCH
SUBROUTINE G13FFF (NUM, NT, IP, IQ, THETA, GAMMA, CVAR, HT, ET, IFLAG)

EGARCH
SUBROUTINE G13FHF (NUM, NT, IP, IQ, THETA, CVAR, HT, ET, IFLAG)

The routine parameters have the following meanings:

Parameters

NUM - INTEGER.
On entry, the number of terms in the arrays *HT* and *ET* from the modelled sequence.

NT - INTEGER.
On entry, the forecast horizon, c

IP - INTEGER.
On entry, the number of moving average coefficients, *p*

IQ - INTEGER.
On entry, the number of auto-regressive coefficients, *q*

THETA - DOUBLE PRECISION array.
On entry, model parameters of the GARCH sequence.

GAMMA - DOUBLE PRECISION.
On entry, the asymmetry parameter g or the GARCH sequence.

CVAR - DOUBLE PRECISION array.
On exit, the forecast expected values of the conditional variance,

HT - DOUBLE PRECISION array.
On entry, the sequence of past conditional variances for the GARCH(*p*, *q*) process, $h_i, i = 1, \ldots, T$.

ET - DOUBLE PRECISION array.
On entry, the sequence of past residuals for the GARCH(*p*, *q*) process, $e_i, i = 1, \ldots, T$.

IFLAG - INTEGER.
The error indicator.

4 Examples of usage

In this section we provide complete Fortran 77 source code to illustrate how the GARCH routines can be used in practice. Each example generates a given type of GARCH sequence models this using the appropriate GARCH estimation and then computes a volatility forecast. All the examples consider GARCH sequences having both Gaussian and Student's *t*-distribution shocks.

4.1 AGARCH-type1

In this example the following two models are considered:

- An AARCH(3)-type1 model with shocks from a Gaussian distribution and observations, $y_t = b_0 + e_t$
- An AGARCH(1,2) -type2 model with shocks from a Student's *t*-distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$

Sequences of 1500 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

Fortran source code

```
      INTEGER NPARMX,NUM
      DOUBLE PRECISION ZERO
      PARAMETER (NPARMX=10,NUM=1500,ZERO=0.0D0)
      INTEGER NUM1,NREGMX,MXNT,NT
      PARAMETER (NUM1=3000,NREGMX=10,MXNT=400)
      DOUBLE PRECISION FAC1,GAMMA,HP,LGF,MEAN,TOL,XTERM
      INTEGER I,IFLAG,IP,IQ,ISYM,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
      LOGICAL FCALL
      DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
     +                 HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
     +                 RVEC(40),SC(NPARMX),SE(NPARMX), THETA(NPARMX),
     +                 WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
     +                 YT(NUM1+10),CVAR(100)
      LOGICAL COPTS(2)
      CHARACTER*1 DIST
      DOUBLE PRECISION DF
      EXTERNAL E04UEF,G05HKF,G13FAF,G13FBF,G05CBF
      INTRINSIC ABS,DBLE,SIN
      WRITE(*,*)'G13FAF Example Program Results'
      SEED = 111
      NREG = 0
      LDX = NUM1
      BX(1) = 1.5D0
      BX(2) = 2.5D0
      BX(3) = 3.0D0
      MEAN = 3.0D0
      DO 5 I = 1,NUM
          FAC1 = DBLE(I)*0.01D0
          X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
          X(I,2) = 0.5D0 + FAC1*0.1D0
          X(I,3) = 1.0D0
    5 CONTINUE
      ISYM = 1
      MN = 1
      GAMMA = -0.4D0
      IP = 0
      IQ = 3
      PARAM(1) = 0.8D0
      PARAM(2) = 0.6D0
      PARAM(3) = 0.2D0
```

```
      PARAM(4) = 0.1D0
      NPAR = 1 + IQ + IP

      LWK = NREG*NUM+3*NUM+NPAR+ISYM+MN+NREG+403
      FCALL = .TRUE.
      IFLAG = 0
      DIST = 'N'
      CALL G05CBF(SEED)
      CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      IFLAG = -1
      DO 10 I = 1,NUM
          XTERM = ZERO
          DO 15 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
   15     CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
   10 CONTINUE
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      COPTS(1) = .TRUE.
      COPTS(2) = .TRUE.
      MAXIT = 200
      TOL = 1.0D-16
      DO 12 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
   12 CONTINUE
      IF (ISYM.EQ.1) THEN
          THETA(NPAR+ISYM) = GAMMA*0.5D0
      END IF
      IFLAG = 0
      CALL G13FAF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,ISYM,
     +        THETA,SE,SC,COVAR,
     +        NPARMX,HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,
     +        LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Gaussian distribution'
      WRITE(*,*)
      WRITE(*,*)
     + 'Parameter estimates  Standard errors  Correct values'

      DO 33 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(I),
     +            SE(I),PARAM(I)
 33   CONTINUE
      IF (ISYM.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+1),
     +   SE(NPAR+1),GAMMA
      END IF
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1),
     +   SE(NPAR+ISYM+1),MEAN
      END IF
      DO 34 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+MN+I),
     +   SE(NPAR+ISYM+MN+I),BX(I)
 34   CONTINUE
      NT = 4
      CALL G13FBF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      WRITE (*,*)
      DIST = 'T'
      NREG = 2
      MN = 1
      DF = 4.1D0
      IP = 1
      IQ = 2
      ISYM = 1
      GAMMA = -0.2D0
      NPAR = IQ + IP + 1
      LWK = NREG*NUM+3*NUM+NPAR+ISYM+MN+NREG+404
```

```
      PARAM(1) = 0.1D0
      PARAM(2) = 0.2D0
      PARAM(3) = 0.3D0
      PARAM(4) = 0.4D0
      PARAM(5) = 0.1D0
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      CALL G05HKF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      IFLAG = -1
      DO 110 I = 1,NUM
          XTERM = ZERO
          DO 115 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
 115      CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
 110  CONTINUE
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      COPTS(1) = .TRUE.
      COPTS(2) = .TRUE.
      MAXIT = 200
      TOL = 1.0D-16
      DO 112 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
 112  CONTINUE
      THETA(NPAR+ISYM) = GAMMA*0.5D0
      THETA(NPAR+ISYM+1) = DF*0.5D0
      CALL G13FAF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,ISYM,
     +            THETA,SE,SC,COVAR,NPARMX,HP,ETM,HTM,LGF,
     +            COPTS,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Student t-distribution'
      WRITE(*,*)
      WRITE(*,*)
     + 'Parameter estimates  Standard errors  Correct values'
      DO 133 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(I),
     +            SE(I),PARAM(I)
133   CONTINUE
      IF (ISYM.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM),
     +   SE(NPAR+ISYM),GAMMA
      END IF
      WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1),
     +   SE(NPAR+ISYM+1),DF
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1+MN),
     +   SE(NPAR+ISYM+1+MN),MEAN
      END IF
      DO 134 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F13.4)') THETA(NPAR+ISYM+1+MN+I),
     +   SE(NPAR+ISYM+1+MN+I),BX(I)
134   CONTINUE
199   CONTINUE
      NT = 4
      CALL G13FBF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      END
```

Output results

```
 G13FAF Example Program Results

 Gaussian distribution

 Parameter estimates  Standard errors  Correct values
          0.8031           0.0788        0.8000
          0.6249           0.0570        0.6000
          0.1803           0.0327        0.2000
          0.0921           0.0237        0.1000
         -0.5119           0.0682       -0.4000
          2.9860           0.0324        3.0000

Volatility forecast =       2.8040


 Student t-distribution

 Parameter estimates  Standard errors  Correct values
          0.0871           0.0230        0.1000
          0.2174           0.0488        0.2000
          0.2736           0.0820        0.3000
          0.3588           0.0788        0.4000
         -0.3240           0.0598       -0.2000
          4.5173           0.5128        4.1000
          3.0182           0.0431        3.0000
          1.4727           0.0265        1.5000
          2.4640           0.0302        2.5000

Volatility forecast =       0.4133
```

4.2 AGARCH-type2

In this example the following two models are considered:

- An AGARCH(1,1)-type2 model with shocks from a Gaussian distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$
- An AGARCH(1,1)-type2 model with shocks from a Student's *t*-distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$

Sequences of 1500 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

Fortran source code

```
      INTEGER NPARMX,NUM
      DOUBLE PRECISION ZERO
      PARAMETER (NPARMX=10,NUM=1500,ZERO=0.0D0)
      INTEGER NUM1,MXNT,NREGMX,NT
      PARAMETER (NUM1=3000,MXNT=400,NREGMX=10)
      DOUBLE PRECISION FAC1,GAMMA,HP,LGF,MEAN,TOL,XTERM
      INTEGER I,IFLAG,IP,IQ,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
      LOGICAL FCALL
      DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
     +                 HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
     +                 RVEC(40),SC(NPARMX),SE(NPARMX),THETA(NPARMX),
     +                 WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
     +                 YT(NUM1+10),CVAR(100)
      LOGICAL COPTS(2)
      CHARACTER*1 DIST
      DOUBLE PRECISION DF
      EXTERNAL E04UEF,G05HLF,G13FCF,G13FDF,G05CBF
      INTRINSIC ABS,DBLE,SIN
```

```
      WRITE(*,*)'G13FCF Example Program Results'
      SEED = 111
      LDX = NUM1
      BX(1) = 1.5D0
      BX(2) = 2.5D0
      BX(3) = 3.0D0
      MEAN = 3.0D0
      DO 5 I = 1,NUM
          FAC1 = DBLE(I)*0.01D0
          X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
          X(I,2) = 0.5D0 + FAC1*0.1D0
          X(I,3) = 1.0D0
    5 CONTINUE
      MN = 1
      NREG = 2
      GAMMA = -0.4D0
      IP = 1
      IQ = 1
      NPAR = IQ + IP + 1
      LWK = NREG*NUM+3*NUM+NPAR+NREG+MN+404
      PARAM(1) = 0.08D0
      PARAM(2) = 0.2D0
      PARAM(3) = 0.7D0
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      DIST = 'N'
      DF = 4.1D0
      CALL G05HLF(DIST,300,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HLF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      DO 110 I = 1,NUM
          XTERM = ZERO
          DO 120 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
  120     CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
  110 CONTINUE
      IFLAG = -1
      DO 130 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
  130 CONTINUE
      THETA(NPAR+1) = GAMMA*0.5D0
      IF (MN.EQ.1) THEN
          THETA(NPAR+1+MN) = MEAN*0.5D0
      END IF
      DO 135 I = 1,NREG
          THETA(NPAR+1+MN+I) = BX(I)*0.5D0
  135 CONTINUE
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      MAXIT = 50
      TOL = 1.0D-12
      COPTS(1) = .TRUE.
      COPTS(2) = .TRUE.
      CALL G13FCF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
     +            SE,SC,COVAR,NPARMX,
     +            HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Gaussian distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
      DO 33 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
33    CONTINUE
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2),
     +   SE(NPAR+2),MEAN
      END IF
      DO 34 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+MN+1+I),
     +   SE(NPAR+MN+1+I),BX(I)
34    CONTINUE
```

```
      NT = 4
      CALL G13FDF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      WRITE (*,*)
      LWK = NUM1*3 + NPARMX + NREGMX*NUM1 + 1
      LDX = NUM1

      BX(1) = 1.5D0
      BX(2) = 2.5D0
      BX(3) = 3.0D0
      MEAN = 3.0D0
      DO 25 I = 1,NUM
          FAC1 = DBLE(I)*0.01D0
          X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
          X(I,2) = 0.5D0 + FAC1*0.1D0
          X(I,3) = 1.0D0
 25   CONTINUE
      MN = 1
      NREG = 2
      GAMMA = -0.4D0
      IP = 1
      IQ = 1
      NPAR = IQ + IP + 1
      LWK = NREG*NUM+3*NUM+NPAR+NREG+MN+405
      PARAM(1) = 0.1D0
      PARAM(2) = 0.1D0
      PARAM(3) = 0.8D0
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      DIST = 'T'
      CALL G05HLF(DIST,300,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HLF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HLF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,HT,YT,
     +            FCALL,RVEC,IFLAG)
      DO 111 I = 1,NUM
          XTERM = ZERO
          DO 121 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
121       CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
111    CONTINUE
      IFLAG = -1
      DO 131 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
131   CONTINUE
      THETA(NPAR+1) = GAMMA*0.5D0
      THETA(NPAR+2) = DF*0.5D0
      IF (MN.EQ.1) THEN
          THETA(NPAR+2+MN) = MEAN*0.5D0
      END IF
      DO 235 I = 1,NREG
          THETA(NPAR+MN+2+I) = BX(I)*0.5D0
 235  CONTINUE
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      MAXIT = 100
      TOL = 1.0D-12
      COPTS(1) = .TRUE.
      COPTS(2) = .TRUE.
      CALL G13FCF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,SE,SC,COVAR,
     +            NPARMX,HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Student t-distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
      DO 133 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
133   CONTINUE
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2),SE(NPAR+2),DF
```

```
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN),
     +   SE(NPAR+2+MN),MEAN
      END IF

      DO 134 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN+I),
     +   SE(NPAR+2+MN+I),BX(I)
 134  CONTINUE
      NT = 4
      CALL G13FDF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      END
```

Output results

```
 G13FCF Example Program Results

 Gaussian distribution

 Parameter estimates  Standard errors  Correct values
          0.0835            0.0154          0.0800
          0.2150            0.0312          0.2000
          0.6896            0.0324          0.7000
         -0.3757            0.0655         -0.4000
          3.0453            0.0591          3.0000
          1.4567            0.0389          1.5000
          2.4572            0.0445          2.5000

Volatility forecast =       3.0383

 Student t-distribution

 Parameter estimates  Standard errors  Correct values
          0.0945            0.0364          0.1000
          0.0800            0.0264          0.1000
          0.8197            0.0523          0.8000
         -0.5142            0.1418         -0.4000
          3.7504            0.3687          4.1000
          3.0045            0.0631          3.0000
          1.5321            0.0378          1.5000
          2.4799            0.0471          2.5000

Volatility forecast =       2.3701
```

4.3 GJR-GARCH

In this example the following two models are considered:

- A GJR-GARCH(1,1) model with shocks from a Gaussian distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$
- A GJR-GARCH(1,1) model with shocks from a Student's *t*-distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$

Sequences of 2000 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

Fortran source code

```
      INTEGER NPARMX,NUM
      DOUBLE PRECISION ZERO
      PARAMETER (NPARMX=10,NUM=2000,ZERO=0.0D0)
      INTEGER NUM1,MXNT,NREGMX,NT
      PARAMETER (NUM1=3000,MXNT=400,NREGMX=10)
      DOUBLE PRECISION FAC1,GAMMA,HP,LGF,MEAN,TOL,XTERM
      INTEGER I,IFLAG,IP,IQ,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
      LOGICAL FCALL
      DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
     +                 HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
     +                 RVEC(40),SC(NPARMX),SE(NPARMX),THETA(NPARMX),
     +                 WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
     +                 YT(NUM1+10),CVAR(100)
      LOGICAL COPTS(2)
      CHARACTER*1 DIST
      DOUBLE PRECISION DF
      EXTERNAL E04UEF,G05HMF,G13FEF,G13FFF,G05CBF
      INTRINSIC ABS,DBLE,SIN

      WRITE(*,*)'G13FEF Example Program Results'
      SEED = 111
      LWK = NUM1*3 + NPARMX + NREGMX*NUM1 + 1
      NREG = 0
      LDX = NUM1
      DF = 5.1D0
      GAMMA = 0.1D0
      BX(1) = 1.5D0
      BX(2) = 2.5D0
      BX(3) = 3.0D0
      MEAN = 4.0D0
      DO 5 I = 1,NUM
          FAC1 = DBLE(I)*0.01D0
          X(I,2) = 0.01D0 + 0.7D0*SIN(FAC1)
          X(I,1) = 0.5D0 + FAC1*0.1D0
          X(I,3) = 1.0D0
    5 CONTINUE
      MN = 1
      NREG = 2
      GAMMA = 0.1D0
      IP = 1
      IQ = 1
      NPAR = IQ + IP + 1
      PARAM(1) = 0.4D0
      PARAM(2) = 0.1D0
      PARAM(3) = 0.7D0
      FCALL = .TRUE.
      DIST = 'N'
      CALL G05CBF(SEED)
      CALL G05HMF(DIST,200,IP,IQ,PARAM,GAMMA,DF,
     +            HT,YT,FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,
     +         HT,YT,FCALL,RVEC,IFLAG)
      DO 76 I = 1,NUM
          XTERM = ZERO
          DO 77 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
   77     CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
   76 CONTINUE
      IFLAG = -1
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      COPTS(1) = .TRUE.
      COPTS(2) = .TRUE.
      MAXIT = 100
      TOL = 1.0D-12
      DO 81 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
   81 CONTINUE
```

```
      THETA(NPAR+1) = GAMMA*0.5D0
      IF (MN.EQ.1) THEN
          THETA(NPAR+MN+1) = MEAN*0.5D0
      END IF
      DO 82 I = 1,NREG
          THETA(NPAR+MN+1+I) = BX(I)*0.5D0
   82 CONTINUE
      CALL G13FEF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
     +         SE,SC,COVAR,NPARMX,
     +         HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Gaussian distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
      DO 33 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
 33   CONTINUE
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2),
     +   SE(NPAR+2),MEAN
      END IF
      DO 34 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+MN+I+1),
     +   SE(NPAR+MN+I+1),BX(I)
 34   CONTINUE
      DIST = 'T'
      MEAN = 3.0D0
      DO 15 I = 1,NUM
          FAC1 = DBLE(I)*0.01D0
          X(I,2) = 0.01D0 + 0.7D0*SIN(FAC1)
          X(I,1) = 0.5D0 + FAC1*0.1D0
          X(I,3) = 1.0D0
   15 CONTINUE
      NT = 4
      CALL G13FFF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      WRITE (*,*)
      MN = 1
      NREG = 2
      GAMMA = 0.09D0
      IP = 1
      IQ = 1
      NPAR = IQ + IP + 1
      PARAM(1) = 0.05D0
      PARAM(2) = 0.1D0
      PARAM(3) = 0.8D0
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      CALL G05HMF(DIST,200,IP,IQ,PARAM,GAMMA,DF,
     +            HT,YT,FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,
     +         HT,YT,FCALL,RVEC,IFLAG)
      CALL G05HMF(DIST,NUM,IP,IQ,PARAM,GAMMA,DF,
     +         HT,YT,FCALL,RVEC,IFLAG)
      DO 176 I = 1,NUM
          XTERM = ZERO
          DO 177 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
  177     CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
  176 CONTINUE
      IFLAG = -1
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      MAXIT = 100
      TOL = 1.0D-14
      DO 181 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
  181 CONTINUE
      THETA(NPAR+1) = GAMMA*0.5D0
      THETA(NPAR+2) = DF*0.5D0
      IF (MN.EQ.1) THEN
```

```
          THETA(NPAR+2+MN) = MEAN*0.5D0
      END IF
      DO 182 I = 1,NREG
          THETA(NPAR+2+MN+I) = BX(I)*0.5D0
  182 CONTINUE
      COPTS(1) = .TRUE.
      COPTS(2) = .TRUE.
      CALL G13FEF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,SE,SC,
     +      COVAR,NPARMX, HP,ETM,HTM,LGF,COPTS,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Student t-distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
      DO 133 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
 133  CONTINUE
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),GAMMA
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2),SE(NPAR+2),DF
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN),
     +   SE(NPAR+2+MN),MEAN
      END IF
      DO 134 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+2+MN+I),
     +   SE(NPAR+2+MN+I),BX(I)
 134  CONTINUE
      NT = 4
      CALL G13FFF(NUM,NT,IP,IQ,THETA,GAMMA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      END
```

Output results

```
 G13FEF Example Program Results

 Gaussian distribution

 Parameter estimates  Standard errors  Correct values
          0.3706            0.0780          0.4000
          0.1034            0.0256          0.1000
          0.7080            0.0413          0.7000
          0.1191            0.0370          0.1000
          4.0989            0.0950          4.0000
          1.4255            0.0592          1.5000
          2.2613            0.0683          2.5000

Volatility forecast =       1.7056


 Student t-distribution

 Parameter estimates  Standard errors  Correct values
          0.0377            0.0084          0.0500
          0.0831            0.0229          0.1000
          0.8112            0.0260          0.8000
          0.1161            0.0361          0.0900
          5.7626            0.6988          5.1000
          2.9674            0.0363          3.0000
          1.4891            0.0231          1.5000
          2.5161            0.0277          2.5000

Volatility forecast =       0.5971
```

4.4 EGARCH

In this example the following two models are considered:

- An EGARCH(1,1) model with shocks from Gaussian distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$
- An EGARCH(1,2) model with shocks from a Student's *t*-distribution and observations, $y_t = b_0 + x_t^1 b_1 + x_t^2 b_2 + e_t$

Sequences of 2000 observations are generated for both of these processes, and then modelled using initial parameter estimates of half the true values. The final model parameter estimates are then output and a four step ahead volatility forecast is calculated.

Fortran source code

```
      INTEGER NPARMX,NUM
      DOUBLE PRECISION ZERO
      PARAMETER (NPARMX=10,NUM=1500,ZERO=0.0D0)
      INTEGER NUM1,NREGMX,MXNT,NT
      PARAMETER (NUM1=3000,NREGMX=10,MXNT=400)
      DOUBLE PRECISION FAC1,HP,LGF,MEAN,TOL,XTERM
      DOUBLE PRECISION DF
      INTEGER I,IFLAG,IP,IQ,K,LDX,LWK,MAXIT,MN,NPAR,NREG,SEED
      LOGICAL FCALL
      CHARACTER*1 DIST
      DOUBLE PRECISION BX(10),COVAR(NPARMX,NPARMX),ETM(NUM1),
     +                 HT(NUM1+10),HTM(NUM1),PARAM(NPARMX),
     +                 RVEC(40),SC(NPARMX),SE(NPARMX),
     +                 THETA(NPARMX),
     +                 WK(NUM1*3+NPARMX+NREGMX*NUM1+20*20+1),X(NUM1,10),
     +                 YT(NUM1+10),CVAR(100)
      LOGICAL COPT
      EXTERNAL E04UEF,G05HNF,G13FGF,G13FHF,G05CBF
      INTRINSIC ABS,DBLE,SIN

      WRITE(*,*)'G13FGF Example Program Results'
      SEED = 111
      LDX = NUM1
      BX(1) = 1.5D0
      BX(2) = 2.5D0
      BX(3) = 3.0D0
      MEAN = 3.0D0
      DO 5 I = 1,NUM
          FAC1 = DBLE(I)*0.01D0
          X(I,1) = 0.01D0 + 0.7D0*SIN(FAC1)
          X(I,2) = 0.5D0 + FAC1*0.1D0
          X(I,3) = 1.0D0
 5    CONTINUE
      NREG = 2
      MN = 1
      IP = 1
      IQ = 1
      NPAR = IP + 2*IQ + 1
      PARAM(1) = 0.1D0
      PARAM(2) = -0.3D0
      PARAM(3) =  0.1D0
      PARAM(4) =  0.9D0
      DF = 5.0D0
      DIST = 'N'
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      CALL G05HNF(DIST,800,IP,IQ,PARAM,DF,HT,YT,FCALL,
     +            RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HNF(DIST,NUM,IP,IQ,PARAM,DF,HT,YT,FCALL,
     +            RVEC,IFLAG)
```

```
      IFLAG = -1
      DO 110 I = 1,NUM
          XTERM = ZERO
          DO 115 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
115        CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
110   CONTINUE
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      COPT = .TRUE.
      MAXIT = 50
      TOL = 1.0D-12
      DO 120 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
120   CONTINUE
      IF (MN.EQ.1) THEN
          THETA(NPAR+MN) = MEAN*0.5D0
      END IF
      DO 130 I = 1,NREG
          THETA(NPAR+MN+I) = BX(I)*0.5D0
130   CONTINUE
      LWK = NREG*NUM + 3*NUM + 3
      CALL G13FGF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
     +            SE,SC,COVAR,NPARMX,
     +            HP,ETM,HTM,LGF,COPT,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Gaussian distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
      WRITE(*,*)
      DO 133 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
133   CONTINUE
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),MEAN
      END IF
      DO 134 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+MN+I),SE(NPAR+MN+I),BX(I)
134   CONTINUE
      NT = 4
      CALL G13FHF(NUM,NT,IP,IQ,THETA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      WRITE (*,*)
      NREG = 2
      MN = 1
      IP = 1
      IQ = 2
      NPAR = IP + 2*IQ + 1
      PARAM(1) =  0.1D0
      PARAM(2) = -0.3D0
      PARAM(3) = -0.1D0
      PARAM(4) =  0.1D0
      PARAM(5) =  0.3D0
      PARAM(6) =  0.7D0
      DF = 10.0D0
      DIST = 'T'
      FCALL = .TRUE.
      CALL G05CBF(SEED)
      CALL G05HNF(DIST,NUM,IP,IQ,PARAM,DF,HT,YT,FCALL,RVEC,IFLAG)
      FCALL = .FALSE.
      CALL G05HNF(DIST,NUM,IP,IQ,PARAM,DF,HT,YT,FCALL,RVEC,IFLAG)
      IFLAG = -1
      DO 10 I = 1,NUM
          XTERM = ZERO
          DO 15 K = 1,NREG
              XTERM = XTERM + X(I,K)*BX(K)
15        CONTINUE
          IF (MN.EQ.1) THEN
              YT(I) = MEAN + XTERM + YT(I)
          ELSE
              YT(I) = XTERM + YT(I)
          END IF
10    CONTINUE
```

```
      CALL E04UEF('Nolist')
      CALL E04UEF('Print Level = 0')
      COPT = .TRUE.
      MAXIT = 50
      TOL = 1.0D-12
      DO 20 I = 1,NPAR
          THETA(I) = PARAM(I)*0.5D0
 20   CONTINUE
      THETA(NPAR+1) = DF*0.5D0
      IF (MN.EQ.1) THEN
          THETA(NPAR+1+MN) = MEAN*0.5D0
      END IF
      DO 30 I = 1,NREG
          THETA(NPAR+1+MN+I) = BX(I)*0.5D0
 30   CONTINUE
      LWK = NREG*NUM + 3*NUM + 3
      CALL G13FGF(DIST,YT,X,LDX,NUM,IP,IQ,NREG,MN,THETA,
     +            SE,SC,COVAR,NPARMX,
     +            HP,ETM,HTM,LGF,COPT,MAXIT,TOL,WK,LWK,IFLAG)
      WRITE(*,*)
      WRITE(*,*)'Student t-distribution'
      WRITE(*,*)
      WRITE(*,*)'Parameter estimates  Standard errors  Correct values'
      DO 33 I = 1, NPAR
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(I),SE(I),PARAM(I)
 33   CONTINUE
      WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1),SE(NPAR+1),DF
      IF (MN.EQ.1) THEN
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1+MN),
     +   SE(NPAR+1+MN),MEAN
      END IF
      DO 34 I = 1, NREG
         WRITE(*,'(F16.4,F18.4,F16.4)') THETA(NPAR+1+MN+I),
     +   SE(NPAR+1+MN+I),BX(I)
 34   CONTINUE
      NT = 4
      CALL G13FHF(NUM,NT,IP,IQ,THETA,CVAR,HTM,ETM,IFLAG)
      WRITE (*,*)
      WRITE (*,'(A,F12.4)') 'Volatility forecast = ',CVAR(NT)
      END
```

Output Results

```
 G13FGF Example Program Results

 Gaussian distribution

 Parameter estimates  Standard errors  Correct values

          0.1153            0.0197          0.1000
         -0.3096            0.0258         -0.3000
          0.1210            0.0382          0.1000
          0.8937            0.0141          0.9000
          2.8624            0.0796          3.0000
          1.4518            0.0445          1.5000
          2.5815            0.0534          2.5000

 Volatility forecast =       2.9468


 Student t-distribution

 Parameter estimates  Standard errors  Correct values
          0.0992            0.0363          0.1000
         -0.2467            0.0548         -0.3000
         -0.1361            0.0529         -0.1000
          0.0670            0.0846          0.1000
          0.3498            0.0680          0.3000
          0.7298            0.0543          0.7000
          7.9630            1.4374         10.0000
          2.9934            0.0734          3.0000
          1.4935            0.0535          1.5000
          2.4933            0.0564          2.5000

Volatility forecast =       1.4868
```

5 Testing the software

The method used here to check whether the GARCH parameter estimation software has been implemented correctly is to use it to model known GARCH sequences.

In each test a known regression-GARCH(*p,q*) process is defined for a fixed parameter vector q . Monte Carlo simulation is then used to repeatedly generate many different GARCH sequences and compute the mean and variance of the q estimates. In each of the tests:

- the initial parameter estimates are taken to be half the true values
- only GARCH(1,2) sequences are modelled

The time dependent regression vector x_t has components in all the tests has the form:

- a constant component, 1
- a linear ramp component, $\frac{1}{2}+\frac{t}{1000}$
- a sinusoidal component, $\frac{1}{100}+0.7\sin\left(\frac{t}{100}\right)$

The difficulty of modelling a GARCH(*p,q*) sequence depends on both p and q and also on how much *volatility memory* there is in the process. Higher values of the b_i parameters give rise to more volatility memory and are therefore harder to model accurately. Increasing the number of model parameters will also make the model more difficult to model simply because there are more variables to numerically optimise. This suggests the following order of difficulty ARCH(1), ARCH(2), ARCH(3), GARCH(1,1), GARCH(1,2), GARCH(2,2),…,etc.

It can be seen that a GARCH(1,2) model therefore represents a *reasonably* difficult test of the software.

As previously mentioned, the simulations are used to calculate the mean of the parameter estimates q and the mean of the estimated standard errors. These values are then compared with the true model parameter values and the actual standard errors of the parameter values.

The simulation results are presented in tables 1-20 of Section 6. The first column labelled "Estimated Value" refers to the average parameter estimate using either 300 or 200 simulations. The second column labelled "Estimated Standard Error" refers to the average of the standard errors computed by the GARCH software. The third column labelled "Standard Error of Estimates" refers to the actual standard error of the parameter estimates.

6 Monte Carlo results

AGARCH-type1

The AGARCH(1,2)-type1 simulations used here had the following parameters:

$$k = 3$$
$$a_0 = 0.2,\ a_1 = 0.1,\ a_2 = 0.15,\ b_1 = 0.7$$
$$g = -0.2$$
$$b_0 = 0,\ b_1 = -1.5,\ b_2 = 2.5,\ b_3 = -3.0$$
$$x_t^1 = \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right)$$
$$x_t^2 = \frac{1}{2} + \frac{t}{1000}$$
$$x_t^3 = 1$$

The series shocks were taken from either a Gaussian distribution or a Student's *t*-distribution, with the number of degrees of freedom (df) set to 4.10.
The GARCH model parameters are output in the descending order, a_0, a_1, a_2, b_1, g, then df (if present) followed by $b_i, i = 1,\ldots,3$

Gaussian distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2267	0.1712	0.1147	0.20
0.0873	0.0581	0.0607	0.10
0.1483	0.0903	0.0816	0.15
0.6944	0.1046	0.0793	0.70
-0.2408	0.3024	0.2398	-0.20
-1.4982	0.2586	0.2472	-1.50
2.5562	0.8595	0.7728	2.50
-3.0455	0.6588	0.5912	-3.00

Table 1: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2022	0.0669	0.0639	0.20
0.0924	0.0411	0.0396	0.10
0.1468	0.0572	0.0526	0.15
0.7050	0.0517	0.0473	0.70
-0.2109	0.1598	0.1380	-0.20
-1.5072	0.0962	0.0971	-1.50
2.5011	0.1619	0.1571	2.50
-3.0001	0.1695	0.1651	-3.00

Table 2: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2038	0.0467	0.0442	0.20
0.0959	0.0285	0.0283	0.10
0.1526	0.0395	0.0378	0.15
0.6986	0.0357	0.0332	0.70
-0.2076	0.0991	0.0944	-0.20
-1.5004	0.0708	0.0661	-1.50
2.4993	0.0576	0.0559	2.50
-2.9998	0.0952	0.0902	-3.00

Table 3: 2000 observations and 200 simulations

Student's *t*-distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1983	0.0706	0.0676	0.20
0.0937	0.0496	0.0592	0.10
0.1528	0.0684	0.0765	0.15
0.6970	0.0586	0.0582	0.70
-0.2321	0.1954	0.1850	-0.20
4.3246	0.6645	0.6189	4.10
-1.5000	0.0765	0.0812	-1.50
2.4907	0.1270	0.1274	2.00
-2.9950	0.1331	0.1354	-3.00

Table 4: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2011	0.0463	0.0456	0.20
0.0975	0.0369	0.0337	0.10
0.1523	0.0485	0.0472	0.15
0.6954	0.0379	0.0394	0.70
-0.2203	0.1338	0.1213	-0.20
4.2278	0.4241	0.4069	4.10
-1.5064	0.0524	0.0517	-1.50
2.4974	0.0451	0.0439	2.00
-2.9987	0.0743	0.0709	-3.00

Table 5: 2000 observations and 200 simulations

AGARCH-type2

The AGARCH(1,2)-type2 simulations used here had the following parameters:

$$k = 3$$
$$a_0 = 0.1, a_1 = 0.1, a_2 = 0.15, b_1 = 0.7$$
$$g = -0.1$$
$$b_0 = 0, b_1 = -1.5, b_2 = 2.5, b_3 = -3.0$$
$$x_t^1 = \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right)$$
$$x_t^2 = \frac{1}{2} + \frac{t}{1000}$$
$$x_t^3 = 1$$

The series shocks were taken from either a Gaussian distribution or a Student's *t*-distribution, with the number of degrees of freedom (df) set to 4.10.
The GARCH model parameters are output in the descending order, a_0, a_1, a_2, b_1, g, then df (if present) followed by $b_i, i = 1, \ldots, 3$

Gaussian distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1238	0.0651	0.0572	0.10
0.1020	0.0636	0.0627	0.10
0.1459	0.0847	0.0856	0.15
0.6778	0.0914	0.0845	0.70
-0.1097	0.1298	0.1116	-0.10
-1.5245	0.1900	0.1763	-1.50
2.4532	0.6022	0.5550	2.50
-2.9564	0.4586	0.4268	-3.00

Table 6: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1081	0.0324	0.0315	0.10
0.1015	0.0436	0.0402	0.10
0.1482	0.0587	0.0538	0.15
0.6918	0.0441	0.0487	0.70
-0.0993	0.0669	0.0613	-0.10
-1.5029	0.0678	0.0675	-1.50
2.4971	0.1083	0.1089	2.50
-2.9935	0.1116	0.1142	-3.00

Table 7: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1048	0.0219	0.0215	0.10
0.1004	0.0310	0.0285	0.10
0.1491	0.0402	0.0379	0.15
0.6960	0.0329	0.0333	0.70
-0.1017	0.0461	0.0425	-0.10
-1.5006	0.0473	0.0459	-1.50
2.4991	0.0356	0.0391	2.50
-2.9996	0.0571	0.0628	-3.00

Table 8: 2000 observations and 200 simulations

Student's *t*-distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1103	0.0397	0.0344	0.10
0.0984	0.0548	0.0568	0.10
0.1547	0.0746	0.0752	0.15
0.6856	0.0666	0.0598	0.70
-0.1060	0.0942	0.0848	-0.10
4.2912	0.6639	0.6037	4.10
-1.4998	0.0460	0.0540	-1.50
2.5087	0.0853	0.0882	2.00
-3.0118	0.0860	0.0931	-3.00

Table 9: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1035	0.0236	0.0221	0.10
0.1005	0.0360	0.0347	0.10
0.1528	0.0488	0.0484	0.15
0.6924	0.0417	0.0404	0.70
-0.0998	0.0599	0.0569	-0.10
4.1854	0.4345	0.3988	4.10
-1.4971	0.0324	0.0360	-1.50
2.4986	0.0284	0.0306	2.00
-2.9977	0.0441	0.0494	-3.00

Table 10: 2000 observations and 200 simulations

GJR-GARCH

The GJR-GARCH(1,2) simulations used here had the following parameters:

$$k = 3$$
$$\alpha_0 = 0.1, \alpha_1 = 0.15, \alpha_2 = 0.2, \beta_1 = 0.4$$
$$\gamma = 0.1$$
$$b_0 = 0, b_1 = -1.5, b_2 = 2.5, b_3 = -3.0$$
$$x_t^1 = \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right)$$
$$x_t^2 = \frac{1}{2} + \frac{t}{1000}$$
$$x_t^3 = 1$$

The series shocks were taken from either a Gaussian distribution or a Student's *t*-distribution, with the number of degrees of freedom (df) set to 4.10.
The GARCH model parameters are output in the descending order, $\alpha_0, \alpha_1, \alpha_2, \beta_1, \gamma$, then df (if present) followed by $b_i, i = 1,\dots,3$

Gaussian Distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1103	0.0410	0.0364	0.10
0.1367	0.0766	0.0774	0.15
0.2003	0.1185	0.1045	0.20
0.3805	0.1435	0.1270	0.40
0.1006	0.0766	0.0759	0.10
-1.4983	0.1010	0.0992	-1.50
2.4982	0.3288	0.3119	2.50
-3.0025	0.2560	0.2397	-3.00

Table 11: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1048	0.0269	0.0221	0.10
0.1451	0.0537	0.0502	0.15
0.1997	0.0752	0.0669	0.20
0.3936	0.0927	0.0784	0.40
0.0958	0.0511	0.0478	0.10
-1.5038	0.0379	0.0387	-1.50
2.4975	0.0645	0.0627	2.50
-2.9970	0.0677	0.0659	-3.00

Table 12: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1018	0.0172	0.0152	0.10
0.1470	0.0387	0.0358	0.15
0.2001	0.0499	0.0476	0.20
0.3972	0.0602	0.0548	0.40
0.1007	0.0385	0.0340	0.10
-1.5016	0.0261	0.0264	-1.50
2.4998	0.0229	0.0224	2.50
-3.0000	0.0363	0.0361	-3.00

Table 13: 2000 observations and 200 simulations

Student's *t*-distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1027	0.0266	0.0285	0.10
0.1392	0.0691	0.0813	0.15
0.2047	0.0932	0.1028	0.20
0.3904	0.0976	0.1131	0.40
0.1158	0.0655	0.1067	0.10
4.2716	0.6603	0.6395	4.10
-1.5013	0.0328	0.0336	-1.50
2.4977	0.0526	0.0564	2.00
-2.9989	0.0549	0.0599	-3.00

Table 14: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.1008	0.0178	0.0170	0.10
0.1469	0.0501	0.0531	0.15
0.2016	0.0716	0.0685	0.20
0.3940	0.0683	0.0663	0.60
0.1067	0.0489	0.0606	0.10
4.2076	0.4360	0.4040	4.10
-1.4982	0.0221	0.0231	-1.50
2.4994	0.0187	0.0188	2.00
-3.0003	0.0299	0.0314	-3.00

Table 15: 2000 observation and 200 simulations

EGARCH

The EGARCH(1,2) simulations used here had the following parameters:

$$k = 3, a_0 = 0.3, a_1 = -0.2, a_2 = -0.25,$$
$$f_1 = 0.1, f_2 = 0.15, b_1 = 0.3,$$
$$b_0 = 0, b_1 = 1.5, b_2 = 2.5, b_3 = 3.0$$
$$x_t^2 = \frac{1}{100} + 0.7 \sin\left(\frac{t}{100}\right)$$
$$x_t^1 = \frac{1}{2} + \frac{t}{1000}$$
$$x_t^3 = 1$$

The series shocks were taken from either a Gaussian distribution or a Student's *t*-distribution, with the number of degrees of freedom (df) set to 4.10.

The GARCH model parameters are output in the descending order, $a_0, a_1, a_2, f_1, f_2, b_1$, then df (if present) followed by $b_i, i = 1, \ldots, 3$

Gaussian Distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2786	0.1016	0.0974	0.30
-0.1965	0.0758	0.0738	-0.20
-0.2481	0.0958	0.0909	-0.25
0.0699	0.1186	0.1228	0.10
0.1346	0.1445	0.1285	0.15
0.3045	0.2213	0.1841	0.30
1.5181	0.1855	0.1928	1.50
2.5622	0.6413	0.6017	2.50
2.9430	0.4929	0.4626	3.00

Table 16: 400 observations and 300 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2958	0.0709	0.0665	0.30
-0.1954	0.0471	0.0462	-0.20
-0.2533	0.0595	0.0571	-0.25
0.0848	0.0768	0.0789	0.15
0.1432	0.0850	0.0812	0.10
0.2914	0.1437	0.1247	0.30
1.4970	0.0747	0.0756	1.50
2.4969	0.1245	0.1231	2.50
3.0004	0.1301	0.1309	3.00

Table 17: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2982	0.0452	0.0471	0.30
-0.1980	0.0317	0.0326	-0.20
-0.2540	0.0381	0.0401	-0.25
0.0932	0.0508	0.0533	0.15
0.1491	0.0591	0.0545	0.10
0.2967	0.0895	0.0870	0.30
1.5017	0.0528	0.0515	1.50
2.4999	0.0442	0.0431	2.50
2.9984	0.0748	0.0702	3.00

Table 18: 2000 observations and 200 simulations

Student's *t*-distribution

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2811	0.1022	0.0924	0.30
-0.2071	0.0628	0.0639	-0.20
-0.2452	0.0786	0.0794	-0.25
0.0865	0.0929	0.0903	0.10
0.1464	0.0932	0.0924	0.15
0.3223	0.1738	0.1526	0.30
4.3119	0.6819	0.4159	4.10
1.4983	0.0668	0.0738	1.50
2.4871	0.1142	0.1210	2.50
3.0094	0.1195	0.1267	3.00

Table 19: 1000 observations and 200 simulations

Estimated Value	Estimated Standard Error	Standard Error of Estimates	Correct Values
0.2859	0.0702	0.0811	0.30
-0.2016	0.0426	0.0493	-0.20
-0.2483	0.0594	0.0706	-0.25
0.0926	0.0652	0.0657	0.10
0.1512	0.0633	0.0801	0.15
0.3154	0.1174	0.1283	0.30
4.2281	0.4348	0.3854	4.10
1.4942	0.0457	0.0539	1.50
2.4976	0.0413	0.0451	2.50
3.0001	0.0681	0.0734	3.00

Table 20: 2000 observations and 200 simulations

Conclusions

The simulation results presented in tables 1-20 indicate that the GARCH generation and estimation Fortran 77 software performs as expected (The forecasting software has been tested elsewhere [9]). It was found that reliable parameter estimates and associated standard errors could only be achieved when at least 300 observations were included in the GARCH model. More *difficult* GARCH models (those with more parameters to estimate or higher volatility memory) were found to require even more observations to obtain consistent parameter estimates.

This report has discussed the current state of GARCH software that has been developed for Mark 20 of the NAG Fortran 77 Library. Since future modifications/improvements may take place it can only be taken as a guide to the GARCH software that will be eventually included in the NAG numerical libraries. Possible future improvements to the software could include the following:

- Changing the GARCH estimation routines so that some of the model parameters can be fixed (i.e., not estimated via maximum likelihood)
- Other non-Gaussian shocks, such as the generalised error distribution, etc.
- Other univariate GARCH models such GARCH-M etc.
- Multivariate GARCH models
- Generalising the GARCH model from regression-GARCH(*p,q*) to ARMA(*p1,q1*)-GARCH(*p2,q2*)

Other possible modifications, concerned with language/user-interface, are as follows:

- The inclusion of some modern Fortran features, such as memory allocation, to improve the user-interface. Since Fortran 77 does not support memory allocation the current GARCH software requires the user to calculate how much workspace is needed. The routines also only allow up to 20 parameters to be estimated; this should be sufficient for most requirements. The NAG C GARCH software, which uses internal memory allocation, does not have these restrictions and is therefore easier to use.
- The creation of Visual Basic wrappers which perform memory allocation and have optional parameters etc. These could, for example, take the form of a GARCH Microsoft Excel add-in which would have the advantage of interactive use and also shield users from the raw Fortran 77 code.
- The creation of a C++ class that contains all the GARCH functions, has optional/default function parameters and also hides workspace, etc.

To conclude, it should be mentioned that the PC and workstation implementations of the NAG numerical libraries permit software developers to call GARCH functions more easily than the corresponding functions from time series packages. Although time series packages may offer the advantage of interactive interfaces it is not easy (or computationally efficient) to call them programmatically as a component of a larger system. The current NAG delivery mechanism is thus well suited to software developers who want to incorporate individual GARCH routines into a new application.

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